Online Appendix: Global Hegemony and Exorbitant Privilege *

Carolin E. Pflueger Pierre Yared

August 25, 2026

B Empirical Robustness

This Section provides additional analyses and robustness of the relationship between the U.S. borrowing advantage and geopolitical risk, shown in Figure 2 in the main paper.

B.1 Data Description

The main left-hand side variable is the equal-weighted average of three-year government bond yields in six advanced economies (Australia, France, Germany, Italy, Japan, and the United Kingdom) minus the three-year U.S. government bond yield in annualized percent. Yields are the last monthly observations from Global Financial Data (GFD tickers IGUSA3D, IGGBR3D, IGAUS3D, IGITA3D, IGFRA3D, IGDEU3D, IGJPN3D). When a country is missing in a given month (Italy and Australia have a few gaps), the AE average is taken over the remaining countries. The German three-year yield is extended after the GFD series ends using Bloomberg (GTDEM3Y Govt), also at month-end. French three-year yields are treated as missing before 1980, when the series becomes reliable. The spread is then stochastically detrended by subtracting a trailing 10-year (120-month) moving average. The main right-hand side variable is geopolitical risk. We use the standardized 12-month moving average of the log U.S. historical geopolitical risk index of Caldara and Iacoviello (2022) (GPRHC USA) to reduce month-to-month noise. The first two columns in Table B.1, Panel A, report summary statistics for these main variables.

In our regressions, we control for a variety of liquidity variables. The Aaa-Treasury spread is the difference between the Aaa Moody's corporate bond yield vs. LTGOVTBD through 1999 and GS20 from 2000 (all from FRED) (Krishnamurthy and Vissing-Jorgensen, 2012). HPW Liquidity is the Treasury bond liquidity measure of Hu, Pan and Wang (2013) available until 2019 via Jun Pan's website (https://en.saif.sjtu.edu.cn/junpan/Noise_Measure.xlsx). CIP denotes the monthly average CIP deviation of U.S. Treasury bonds (3-year tenor) minus the average of the U.K., Australia, Germany, and Japan from Du, Im and Schreger (2018), updated by Du, Keerati and Schreger (2025) (data updated October 2025, available from https://jschreger.s3.us-east-2.amazonaws.com/cip_dataset_v4.dta). The set of countries for CIP differs from our main analysis due to data availability. A positive value for CIP means that Treasuries are more convenient. The Aaa–Treasury spread is in percentage points, whereas HPW and CIP are in basis points. Summary statistics for these liquidity variables are reported in the remaining columns of Table B.1, Panel A.

*Pflueger: University of Chicago, Harris School of Public Policy and NBER, cpflueger@uchicago.edu; Yared: Columbia Business School and NBER, pyared@columbia.edu

Table B.1: Summary Statistics

Panel A: U.S. Borrowing Advantage, Geopolitical Risk, Liquidity

	U.S. Borrowing Advantage	Geopol. Risk	Aaa-Tsy	HPW	CIP
Average	-0.15	-0.01	0.91	3.14	11.82
Std	1.35	1.00	0.32	2.07	19.93
Min	-3.08	-2.87	0.38	0.87	-43.86
Max	3.19	2.29	1.87	18.38	107.70
Obs.	527	527	527	393	287
Period	1980–2023	1980–2023	1980–2023	1987–2019	2000–2023

Panel B: Inflation and Financial Market Condition Controls

	Infl.	Infl. Spread	USD Vol	VIX	Haven Spread
Average	3.55	0.25	4.41	19.63	-0.47
Std	2.23	0.92	0.68	7.35	0.97
Min	1.29	-2.52	3.12	10.13	-4.07
Max	9.94	1.43	6.08	78.34	2.39
Obs.	527	527	527	527	527
Period	1980–2023	1980–2023	1980–2023	1980–2023	1980–2023

Note: This table reports summary statistics for Figure 2 in the main paper from January 1980 until November 2023 and for liquidity, inflation and financial market condition control variables. The sample period for each variable is noted in the bottom row. The U.S. borrowing advantage is defined as the spread between the average three-year government bond yield across advanced economies (the United Kingdom, Australia, Italy, France, Germany, and Japan) and the U.S. three-year government bond yield, expressed as a deviation from its trailing 120-month moving average. Geopolitical risk is the standardized 12-month moving average of the log U.S. geopolitical risk index of Caldara and Iacoviello (2022). Detailed variable descriptions for the control variables are provided in the text in Appendix B.1.

We also control for other determinants of sovereign yield spreads, such as inflation, exchange rate risk, and global financial conditions. U.S. inflation is measured as year-over-year CPI inflation, $\pi_t^{US} = 100 \times (\text{CPI}_t/\text{CPI}_{t-12} - 1)$, where the CPI is the U.S. Consumer Price Index (CPI-U, all items, not seasonally adjusted) from Robert Shiller's website. "Infl." is defined as the trailing five-year (60-month) moving average of π_t^{US}.

The spread of U.S. inflation vs. other advanced economies ("Infl. Spread") is the difference between the five-year moving average of U.S. inflation and the five-year moving average of an advanced-economy (AE) inflation aggregate. The AE aggregate is the equal-weighted cross-country average of year-over-year national CPI inflation for the U.K., Australia, Italy, France, Germany, and Japan, using whichever of the six countries are available in a given month. Non-U.S. CPI data are from Global Financial Data.

To construct exchange rate volatility, we measure the U.S. exchange rate as the broad trade-weighted nominal U.S. dollar index at monthly frequency. We splice the discontinued monthly Broad Index (TWEXBMTH) for January 1973 through December 2005 to the daily Nominal Broad U.S. Dollar Index, Goods and Services (DTWEXBGS), averaged within each month, from January 2006 onward. To make the level continuous, the post-2006 series is rescaled into the units of the pre-2006 series by the ratio of the two indices' January 2006 values. "USD Vol." is defined as the trailing five-year (60-month) standard deviation of monthly changes in the broad dollar index, annualized and expressed in percent. We compute monthly log changes of the spliced index, $\ln(\text{USD}_t) - \ln(\text{USD}_{t-1})$, take their standard deviation over the rolling 60-month window ending in month t, and convert into annualized percent units by multiplying by $100 \times \sqrt{12}$.

We control for financial market conditions using the VIX, which we measure following Nagel (2016) as CBOE option-implied volatility from FRED averaged within each month. It is extended before 1990 with the long VIX series from Nagel (2016). Finally, "Haven Spread" measures the borrowing-cost advantage of non-U.S. economies that are generally considered to have safe haven properties. It is constructed analogously to the U.S. borrowing advantage: the equal-weighted average of 3-year government yields in the remaining AE countries (U.K., Australia, France, and Italy) minus the equal-weighted average of 3-year yields in safe haven countries (Germany, Japan, and Switzerland), then stochastically detrended by subtracting a trailing 10-year moving average. When Swiss yields are missing in early years, we substitute the 5-year yield for the Swiss 3-year yield. If the Swiss 5-year yield is also missing we use the simple average of German and Japanese yields. Summary statistics for inflation, exchange rate volatility, and financial market conditions are reported in Table B.1, Panel B.

B.2 Regressing the U.S. Borrowing Advantage on Geopolitical Risk

We now regress the U.S. borrowing advantage onto geopolitical risk and show that the relationship is robust to liquidity, inflation and financial market conditions. Table B.2 starts with a baseline regression of the U.S. borrowing advantage onto geopolitical risk. Geopolitical risk is standardized, so a one-standard deviation increase in geopolitical risk is associated with a 61 bps increase in the U.S. borrowing advantage. This relationship is highly statistically significant and economically meaningful. Next we control for liquidity variables, the Aaa–Treasury spread as a measure of Treasury convenience from Krishnamurthy and Vissing-Jorgensen (2012), the Treasury bond liquidity variable from Hu, Pan and Wang (2013), and covered interest parity deviations and find that the relationship between the U.S. borrowing advantage and geopolitical risk is robust. A somewhat lower but still statistically significant and economically meaningful coefficient obtains while controlling for relative Treasury convenience (CIP) for a shorter period, starting in 2000. Finally, the last column uses the borrowing cost advantage without detrending, showing that the relationship is not sensitive to de-trending choices.

Panel B includes controls for five-year U.S. average inflation ("Infl."), the spread between five-year U.S. inflation and other advanced economies ("Infl. Spread"), exchange rate volatility, and the VIX as a measure of stock market volatility. Exchange rate volatility has the most economically meaningful impact on the regression R-squared in Panel B, but the coefficient on geopolitical risk remains economically unchanged and statistically significant. The last column controls for a safe haven borrowing spread advantage ("Haven Spread") and finds that the relationship between geopolitical risk and the U.S. borrowing advantage remains economically and statistically significant, suggesting that the relationship between borrowing advantage and geopolitical risk is not driven by a broader time-varying demand for safe havens. Overall, the relationship between the U.S. borrowing advantage and geopolitical risk is economically meaningful, statistically significant, and not absorbed by liquidity, inflation, or financial market conditions.

Table B.2: U.S. Borrowing Advantage onto Geopolitical Risk with Controls

Panel A: Controlling for Liquidity					
	Baseline	Aaa-Tsy	HPW	CIP	Undetrended
Geopolitical risk	0.61***	0.61***	0.74***	0.38**	0.40***
	(0.14)	(0.14)	(0.13)	(0.16)	(0.14)
Control		0.17	0.25***	0.02***	
		(0.50)	(0.07)	(0.01)	
Observations	527	527	393	287	527
R^2	0.20	0.21	0.38	0.18	0.08

Panel B: Controlling for Inflation and Financial Market Conditions					
	Infl.	Infl. Spread	USD Vol	VIX	Haven Spread
Geopolitical risk	0.57***	0.61***	0.62***	0.66***	0.66***
	(0.16)	(0.14)	(0.13)	(0.14)	(0.14)
Control	0.04	0.01	0.80***	0.04***	-0.21
	(0.08)	(0.19)	(0.17)	(0.01)	(0.14)
Observations	527	527	527	527	527
R^2	0.21	0.20	0.36	0.26	0.22

Note: This table reports regressions of the form $\bar{y}_{3,t} - y^{US}_{3,t} = \alpha + \beta \text{GeoRisk}_t + \gamma X_t + \varepsilon_t$, adding one control at a time. All regressions include a constant. The dependent variable is the spread between the average three-year government bond yield across advanced economies (the United Kingdom, Australia, Italy, France, Germany, and Japan) and the U.S. three-year government bond yield, expressed as a deviation from its trailing 120-month moving average. Geopolitical risk is the standardized 12-month moving average of the log U.S. geopolitical risk index of Caldara and Iacoviello (2022). All variables are monthly with summary statistics and sample periods listed in Table B.1. Newey–West (Bartlett) heteroskedasticity- and autocorrelation-robust standard errors with a bandwidth of 18 months are reported in parentheses. The sample runs from January 1980 to November 2023, with the exception of CIP deviations data, which starts in January 2000. Coefficients marked with *, **, and *** are significant at the 10%, 5%, and 1% levels, respectively.

C Endogenous War Probability

In our simple model, the probability of war, ϕ, is set exogenously. Given our Assumption 1, which takes the cost of losing a war to infinity, no rational policy maker would ever choose to initiate a war, since the probability of losing is always positive for both contestants. There is, hence, no reason why in our model the probability of war should be related to the levels of armament. The rationalist approach to the origins of war has emphasized the necessity for bargaining failures and asymmetric information for war to erupt (Fearon, 1995; Powell, 1993, 1999). We abstract from these forces in our model. However, even in the presence of asymmetric information, it would simply be too costly in expectation to go to war in our model.

In reality, of course, war does not occur randomly. In this Section, we provide a simple extension capturing in reduced-form how relative levels of armament may be related to the probability of war. We allow for a flexible functional form for the war probability $\phi = \phi(\mu)$. This allows us to capture the long-standing schools of thought that the probability of war is linked to the "balance of powers", with Morgenthau (1948) and Waltz (1979) proposing that the probability of war should be smallest when powers are balanced. Conversely, the "preponderance of power" school of thought has argued that the probability of war is minimized when there is a predominant power (Organski and Kugler, 1980). Both of these cases can be nested by a function $\phi(\mu)$. However, a functional form $\phi(\mu)$ cannot capture mechanisms whereby the probability of war may increase with total armament (Wallace, 1979; Sample, 1997). Introducing the total level of armament would require adding an additional state variable to our model, and thereby substantially complicate the analysis.

Going forward, we generalize our baseline assumption and allow for a potentially endogenous war probability of the form:

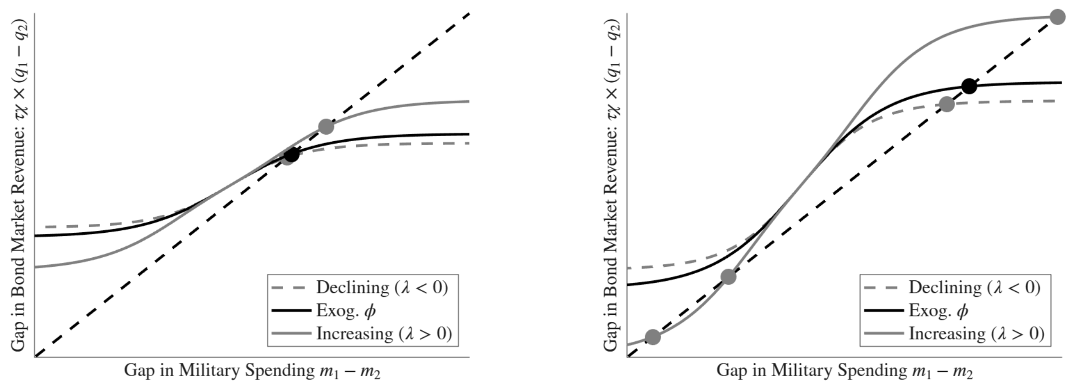
$$\phi\left(m_1, m_2\right) \quad = \quad \bar{\phi}\left(1+\lambda\left(2F\left(A+m_1-m_2\right)-1\right)^2\right), \tag{C.1}$$

where λ is a scaling parameter that can be positive or negative. Note that $\left(2F\left(A+m_1-m_2\right)-1\right)^2 = \left(w_1(m_1,m_2)-w_2(m1,m_2)\right)^2$ is simply the difference between the winning probabilities squared. The squared specification ensures that the war probability can either increase when the countries are most unequal ($\lambda > 0$) or when countries are most similar ($\lambda < 0$). The key new parameter is λ. When this parameter is positive, a greater imbalance in power makes war more likely and more balanced powers minimize the probability of war breaking out ("balance of powers" scenario). Conversely, when λ is negative, a strongly dominant country makes it less likely that war will break out ("preponderance of power" scenario). Beyond $\lambda > -1$ and $\bar{\phi}(1+\lambda) < 1$, which keep $\phi(m_1, m_2) \in (0, 1)$, we restrict $\lambda \in (-0.2, 3)$ so that each country's unconditional probability of defeat, $\phi(m_1, m_2)\left(1 - w_i(m_i, m_{-i})\right)$, is decreasing in its own military spending irrespective of A, $\bar{\phi}$, and the units of military capacity. This ensures that the corner solution for non-military consumption and debt remains optimal.

Figure C.1 shows how the unique equilibrium case in the two-period model changes as we allow for an endogenous war probability. Figure C.1, Panel A, plots the "$A \approx 0$, Low χ" curve of Figure 3A in solid black, together with versions that have $\lambda > 0$ and $\lambda < 0$, respectively. As before, an equilibrium is a point at which a revenue curve intersects the dashed 45-degree line, marked by a circle. Panel A shows that when more unequal powers are more likely to make the mistake of going to war ($\lambda > 0$), equilibria become more extreme, with the dominant country commanding a stronger advantage. Relative to the constant-war-probability benchmark ($\lambda = 0$), the $\lambda > 0$ equilibrium features a strictly larger gap in both military spending and bond market revenue in favor of the stronger country, while the $\lambda < 0$ equilibrium is more compressed. The intuition is that when a larger gap in strength raises the war probability, the feedback mechanism through bond

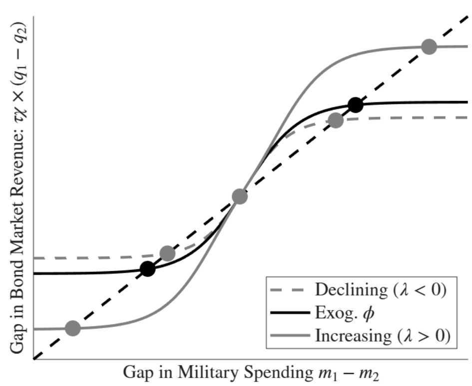
Figure C.1: Endogenous War Extension in Two-Period Model: Unique Equilibrium

Panel A: $A \approx 0$, Low χ **Panel B:** High A, High χ



This figure is a robustness version of Figure 3A in the main paper. Different from Figure 3A in the main paper, the war probability here is endogenous to the military imbalance. We let the war probability equal its constant baseline scaled by the factor $1+\lambda\,(2F(A+m_1-m_2)-1)^2$, where λ governs how much more likely conflict becomes as powers grow more unequal. The gap in military spending $m_1 - m_2$ is on the x-axis and the gap in bond market revenue $\tau\chi \times (q_1 - q_2)$ is on the y-axis; an equilibrium is the intersection of a revenue curve with the dotted 45-degree line (circles). Each panel plots three curves: $\lambda = 0$ (solid black) reproduces exactly the constant-war-probability curve of Figure 3A in the main paper, while $\lambda > 0$ (solid gray) and $\lambda < 0$ (dashed gray) make conflict respectively more and less likely as the imbalance grows. Panel A corresponds to the black curve of Figure 3A and Panel B to the gray curve. The two panels share identical axes.

Figure C.2: Endogenous War Extension in Two-Period Model: Multiple Equilibria



This figure is a robustness version of Figure 3B in the main paper ($A \approx 0$, high χ) in which the war probability is endogenous to the military imbalance, using the same specification as Figure C.1: the war probability equals its constant baseline scaled by $1+\lambda\,(2F(A+m_1-m_2)-1)^2$. The gap in military spending $m_1 - m_2$ is on the x-axis and the gap in bond market revenue $\tau\chi \times (q_1 - q_2)$ is on the y-axis; equilibria are intersections of a revenue curve with the dotted 45-degree line (circles). The curve $\lambda = 0$ (solid black) reproduces exactly the constant-war-probability curve of Figure 3B in the main paper, while $\lambda > 0$ (solid gray) and $\lambda < 0$ (dashed gray) make conflict respectively more and less likely as the imbalance grows.

market revenue is amplified.

Panel B in Figure C.1 shows how the "High A, High χ" case in Figure 3A changes, when we allow for an endogenous war probability. Similar to Panel A, we again see that $\lambda > 0$ makes the equilibrium where country 1 dominates more extreme, while $\lambda < 0$ makes country 1 less dominant in this equilibrium. In addition, we also see that multiple equilibria now emerge more easily when $\lambda > 0$. Holding the exogenous advantage A fixed and holding the war probability at equal effective military advantage fixed, the high-A, high-χ case in Panel B of Figure C.1—which is unique under a constant war probability—develops an additional pair of equilibria once $\lambda > 0$. The mechanism is straightforward. With $\lambda > 0$ and unequal powers, the war probability ϕ rises, and because financial markets price ϕ, this amplifies the feedback loop running from bond market revenue to unequal armaments and back, steepening the revenue curve until it crosses the 45-degree line multiple times.

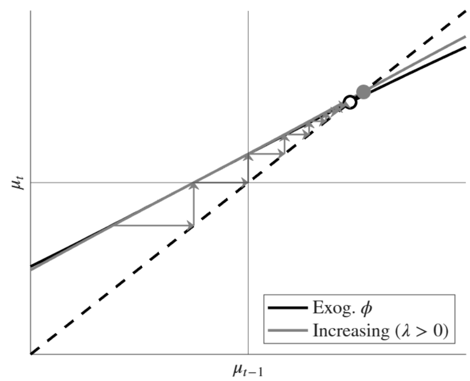
Figure C.2 provides robustness for Figure 3B in the main paper, when we allow for an endogenous war probability. It plots the gap in military spending $m_1 - m_2$ on the x-axis against the gap in bond market revenue $\tau\chi \times (q_1 - q_2)$ on the y-axis for the high-debt-capacity, near-symmetric case ($A \approx 0$, high χ). As before, the solid black curve ($\lambda = 0$) coincides exactly with the constant-war-probability case in the main paper, while the solid gray curve ($\lambda > 0$) and dashed gray curve ($\lambda < 0$) make conflict respectively more and less likely as the military imbalance widens. In this case, each of the three curves crosses the 45-degree line three times, so multiple equilibria are preserved under both $\lambda > 0$ and $\lambda < 0$. The role of λ is to shift how extreme the two dominance equilibria are while leaving multiplicity intact. When $\lambda < 0$, so that conflict becomes less likely as powers grow more unequal, the two outer equilibria move inward and become less extreme—the dominant country's advantage in both armaments and bond market revenue shrinks. When $\lambda > 0$, so that more unequal powers are more prone to the mistake of going to war, the two outer equilibria move outward and become more extreme, with the dominant country commanding a larger gap in both military spending and bond market revenue.

Figures C.3 and C.4 show that the three dynamic cases, uniqueness, hysteresis and fragility, also carry over to the extension with an endogenous probability of war. We again use the functional form (C.1). Figure C.3 is analogous to Figure 4 in the main paper but assumes that the war probability is smallest when military powers are balanced ($\lambda > 0$). Figure C.4 is analogous to Figure 4 in the main paper but assumes that the war probability is smallest when there is one dominant power ($\lambda < 0$). The original mapping with an exogenous and constant ϕ is shown in black and the mapping with an endogenous war probability is shown in gray in each panel. A transition path is depicted for the endogenous war probability case. Uniqueness, hysteresis, and fragility are preserved in each case. Moreover, Figure C.3 shows that in the case where the war probability is minimized when military power is balanced, the steady states shift out in all three panels. In a steady state with a dominant country, dominance is now amplified through the feedback channel from the probability of starting a war compared to our baseline, shown in black. Moreover, Figure C.3, Panel C shows that the zone of fragility is widened when the war probability increases in the military gap between the two countries.

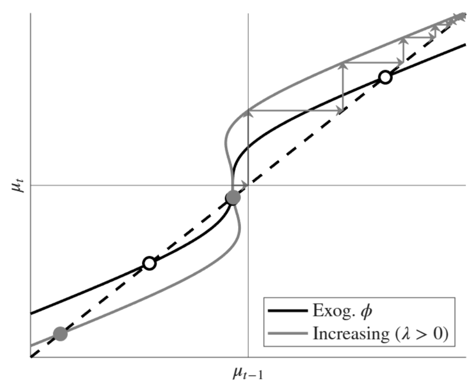
Overall, allowing for an endogenous war probability makes the relationship between debt capacity and multiplicity more complicated, but the main result is preserved. There is a threshold debt capacity, above which multiple steady states emerge, and a second threshold above which geopolitical fragility emerges. Moreover, under the "balance of power" hypothesis that more unequal powers are more likely to lead to war, military and financial dominance in steady state is amplified, and multiplicity and fragility tend to emerge at lower debt capacity than in our baseline with exogenous war probability.

Figure C.3: Endogenous War Probability in Dynamic Model: Balance of Power Case

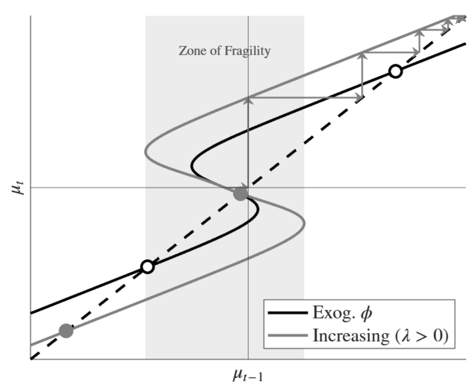
Panel A: Uniqueness



Panel B: Hysteresis



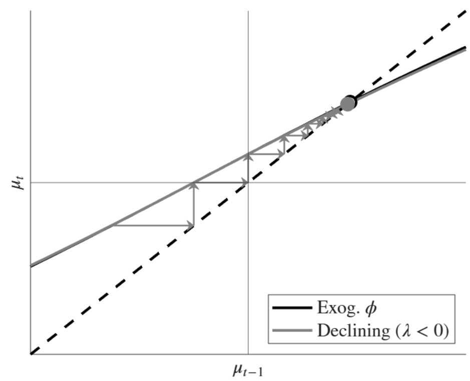
Panel C: Fragility



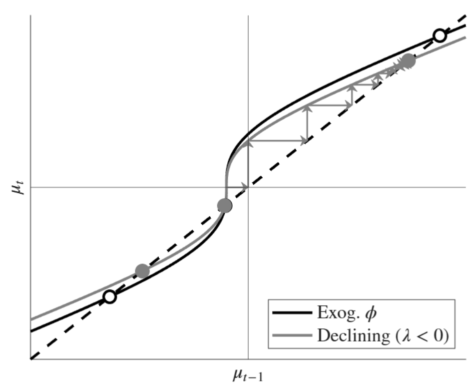
This figure is analogous to Figure 4 in the main paper, but allows for a war probability that increases in the magnitude of the military imbalance ($\lambda > 0$).

Figure C.4: Endogenous War Probability in Dynamic Model: Preponderance of Power Case

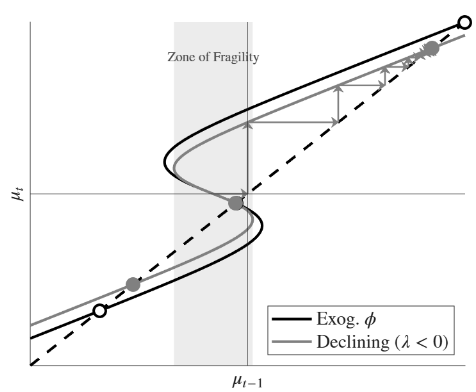
Panel A: Uniqueness



Panel B: Hysteresis



Panel C: Fragility



This figure is analogous to Figure 4 in the main paper, but allows for a war probability that decreases in the magnitude of the military imbalance ($\lambda < 0$).

D Two-Period Model with Concave Utility and Finite Cost

In this Section, we present a two-period model with concave utility and finite cost of losing the war. We show that under certain conditions, all the equilibrium implications from the linear two-period model in the main paper are preserved. The key insight is that at an equilibrium with interior consumption and debt at the corner, both countries endogenously choose the same level of non-military consumption. This leaves the difference in military spending (equation (5) in the main paper) invariant. While the predictions in the model with curvature are necessarily specific to some of the modeling assumptions, this model extension nonetheless shows that linear utility and an infinite cost of war in the main paper are not crucial for our results. We first describe the environment and equilibrium conditions for the two-period model with concave utility. We then prove the main results.

D.1 Environment

There are two countries $i = 1, 2$ and two time periods $t = 0, 1$, where each country is of mass 1. There are risk neutral global investors with deep pockets. For simplicity, we will assume the two countries are identical in every dimension other than military technology whereby one country has a higher chance of winning a war under symmetric levels of military spending.

The date 0 government budget constraint of country i is

$$g_{i0} + m_i = \tau + q_i b_i$$

where g_{i0} is non-military government consumption, m_i is military spending, $\tau > 0$ is an exogenous tax revenue, q_i is the global bond price for country i bonds, and $b_i \geq 0$ is the level of borrowing. Non-military consumption is assumed to be bounded below by 0 and above by an upper bound $\bar{G}$. The date 1 government budget constraint of country i under peace is

$$g_{i1} = \tau (1 - \chi d_i) - b_i (1 - d_i)$$

where $d_i \in \{0, 1\}$ is country i's default decision and $1 > \chi > 0$ is a default cost which is proportional to the tax revenue of the government.

The date 1 budget constraint under war if the country wins is identical to that under peace. However, if the country loses the war, the tax endowment is diminished by some fraction $0 < \gamma < 1$ and the budget constraint becomes

$$g_{i1} = \tau (1 - \gamma) (1 - \chi d_i) - b_i (1 - d_i).$$

The preferences of the country take a standard log form

$$E \{\log g_{i0} + \kappa \log g_{i1}\}$$

where κ takes a value of 1 under peace and a value of $\theta > 1$ under war, capturing the fact that marginal utility of resources under war rises for everyone. Parameters γ and θ can be viewed as two different disaster parameters: θ captures the rise in marginal utility of resources that international investors experience, and γ captures the differential loss of resources for the loser of the war.

International investors are risk neutral with deep pockets with preferences

$$E \{c_0 + \kappa c_1\},$$

where c_t represents the consumption of investors. Global investors are risk neutral, though they do weigh resources more in the event of future war, like the country.

The probability of war is exogenous and equal to ϕ. Country i's probability of winning the war is $w_i(m_i, m_{-i})$ which is increasing in m_i and decreasing in m_{-i}. We consider the difference contest function of Hirshleifer (1989) as in the main text

$$w_1(m_1, m_2) = \frac{exp(A + m_1)}{exp(A + m_1) + exp(m_2)} \equiv F(A + m_1 - m_2). \tag{D.1}$$

The within-period timing is identical to the two-period model in the main text.

D.2 Equilibrium

We solve the program by backward induction. At date 1, under peace or war with victory, country i defaults if

$$b_i > \tau\chi,$$

and under war with loss, country i defaults if

$$b_i > \tau\chi(1-\gamma).$$

This implies that bond prices at date 0, which depend on the probability of war and the probability of victory as well as the level of debt, satisfy the following condition

$$q_i(b_i, m_i, m_{-i}) = \begin{cases} 1 - \phi + \phi\theta & \text{if } b_i \le \tau\chi(1-\gamma) \\ 1 - \phi + \phi\theta w_i(m_i, m_{-i}) & \text{if } b_i \in (\tau\chi(1-\gamma), \tau\chi] \\ 0 & \text{if } b_i > \tau\chi \end{cases}. \tag{D.2}$$

Since choosing $b_i > \tau\chi$ is dominated by the choice $b_i = 0$ for the country, levels of debt which exceed $\tau\chi$ need not be considered by the country. The country therefore decides between a lower level of debt on which it never defaults or a higher level of debt on which it only defaults under war with loss. We can characterize the country's optimization problem in each scenario. Throughout, we use the period 0 budget constraint to substitute for g_{i0} and write the government's problem in terms of (m_i, b_i).

Suppose that the country chooses a value of debt $b_i \le \tau\chi(1-\gamma)$. Then its optimization program is

$$\max_{m_i, b_i \in [0, \tau\chi(1-\gamma)]} \left\{ \begin{array}{c} \log(\tau + (1 - \phi + \phi\theta) b_i - m_i) \\ + (1 - \phi + \phi\theta w_i(m_i, m_{-i})) \log(\tau - b_i) \\ + \phi\theta(1 - w_i(m_i, m_{-i})) \log(\tau(1-\gamma) - b_i) \end{array} \right\}$$

Interior first order conditions in this case are:

$$m_i \quad : \quad \frac{1}{\tau + (1 - \phi + \phi\theta) b_i - m_i} = \phi\theta \frac{\partial w_i(m_i, m_{-i})}{\partial m_i} [\log(\tau - b_i) - \log(\tau(1-\gamma) - b_i)] \tag{D.3}$$

$$b_i \quad : \quad \frac{1 - \phi + \phi\theta}{\tau + (1 - \phi + \phi\theta) b_i - m_i} = (1 - \phi + \phi\theta w_i(m_i, m_{-i})) \frac{1}{\tau - b_i} + \phi\theta(1 - w_i(m_i, m_{-i})) \frac{1}{\tau(1-\gamma) - b_i} \tag{D.4}$$

At the corners for g_{i0} and b_i, the conditions (D.3) and (D.4) turn into weak inequalities.

The second scenario to consider is the one in which the country chooses a value of debt $b_i \in$

$(\tau\chi(1-\gamma), \tau\chi]$. Now the optimization program becomes

$$\max_{m_i, b_i \in (\tau\chi(1-\gamma), \tau\chi]} \left\{ \begin{array}{c} \log\left(\tau + \left(1-\phi+\phi\theta w_i\left(m_i, m_{-i}\right)\right) b_i - m_i\right) \\ +\left(1-\phi+\phi\theta w_i\left(m_i, m_{-i}\right)\right) \log\left(\tau - b_i\right) \\ +\phi\theta\left(1-w_i\left(m_i, m_{-i}\right)\right) \log\left(\tau\left(1-\gamma\right)\left(1-\chi\right)\right) \end{array} \right\}$$

Interior first order conditions in this case are:

$$\begin{aligned} m_i \quad &: \quad \frac{1}{\tau + \left(1-\phi+\phi\theta w_i\left(m_i, m_{-i}\right)\right) b_i - m_i} \left(1 - \phi\theta \frac{\partial w_i\left(m_i, m_{-i}\right)}{\partial m_i} b_i\right) = \\ & \quad\quad \phi\theta \frac{\partial w_i\left(m_i, m_{-i}\right)}{\partial m_i} \left[\log\left(\tau - b_i\right) - \log\left(\tau\left(1-\gamma\right)\left(1-\chi\right)\right)\right] \qquad \text{(D.5)} \\ b_i \quad &: \quad \frac{1-\phi+\phi\theta w_i\left(m_i, m_{-i}\right)}{\tau + \left(1-\phi+\phi\theta w_i\left(m_i, m_{-i}\right)\right) b_i - m_i} = \left(1-\phi+\phi\theta w_i\left(m_i, m_{-i}\right)\right) \frac{1}{\tau - b_i} \qquad \text{(D.6)} \end{aligned}$$

At the corners for g_{i0} and b_i, the conditions (D.5) and (D.6) turn into weak inequalities. Note that now the government internalizes the effect of military spending on the bond price.

D.3 Non-Military Consumption at the Debt Corner

We now describe the properties of equilibria where both countries choose interior non-military consumption and the maximal sustainable debt level, $b_1 = b_2 = \tau\chi$. For the chosen contest function $\frac{\partial w_1}{\partial m_1} = \frac{\partial w_2}{\partial m_2} = w_1 w_2$ is symmetric. Therefore, the levels of the winning probabilities of the two countries are different, but the slopes with respect to own military spending are the same. We hence obtain the following result:

Proposition D.1 *Suppose $b_1 = b_2 = \tau\chi$ and that non-military consumption is interior for both countries. Then both governments choose the same level of non-military consumption with*

$$g_{10} = g_{20} = \frac{1 - \phi\theta\tau\chi \times w_1 w_2}{\phi\theta \times w_1 w_2 \left[-\log(1-\gamma)\right]}. \qquad \text{(D.7)}$$

Proof. Substitute $\frac{\partial w_i}{\partial m_i} = w_1 w_2$ and $b_i = \tau\chi$ into the first-order condition (D.5) and solve for $g_{i0} = \tau + (1-\phi+\phi\theta w_i(m_i, m_{-i}))b_i - m_i$. ■

Corollary D.1 (Military advantage equals borrowing advantage) *Under the conditions of Proposition D.1,*

$$m_1 - m_2 = \left(q_1 - q_2\right)\tau\chi = \phi\theta\tau\chi\left(w_1 - w_2\right). \qquad \text{(D.8)}$$

Proof. Difference the date 0 budget constraints $g_{i0} = \tau + q_i\tau\chi - m_i$ and use $g_{10} = g_{20}$ together with $q_i = 1 - \phi + \phi\theta w_i$. ■

Corollary D.1 is the concave-utility counterpart of equation (5) in the main paper and says that the funding advantage funds relative military capacity in equilibrium. Different from the linear model in the main paper, non-military consumption here does respond to funding conditions and the military balance. However, because the marginal effect of military spending for the probability of winning is the same for both countries, both countries choose the same endogenous level of non-military consumption. Taking the difference, the relationship between funding advantage and relative military spending is preserved.

D.4 Convergence to Corner Case

So far, we have characterized the properties of equilibria where non-military consumption is interior but debt is at the upper corner. We now show that for a sufficiently large but finite cost of losing, any equilibrium indeed takes this form.

Proposition D.2 *Assume that $\phi\theta\tau\chi < 4$. There exists a threshold $\gamma' < 1$ such that for $\gamma > \gamma'$, both governments optimally set $b_i = \tau\chi$ and $g_{10} = g_{20} < \bar{G}$.*

With Proposition D.1 and Corollary D.1, it then follows that the equilibria for the relative military balance $m_1 - m_2$ and the associated borrowing cost differential are identical to the two-period linear model in the main paper. The condition $\phi\theta\tau\chi < 4$ ensures that there is a trade-off between military spending and non-military consumption, conditional on the other country's strategy, and that the numerator of (D.7) is positive. It is a sufficient but not necessary condition. Importantly, it does not constrain debt capacity to be either below or above the multiplicity threshold, χ', and hence both uniqueness and multiplicity are possible.

Proof. Note that because g_{i0} and q_i are both bounded above and below, m_i is also bounded above and below via the budget constraint. It follows that $w_i(m_i, m_{-i})$ is bounded away from 0 and 1 and $\frac{\partial w_i}{\partial m_i}$ is bounded away from zero. Let $\underline{\omega}$ denote the resulting lower bound on $\frac{\partial w_i}{\partial m_i}$.

First, we show that we are in the case with $g_{i0} < \bar{G}$, i.e. the upper bound on non-military consumption is slack. Suppose not. There exists a threshold $\gamma_1 < 1$ such that if $\gamma > \gamma_1$ the right-hand sides of (D.3) and (D.5) exceed $\frac{1}{\bar{G}}$, which is a contradiction. We are hence in the case with $g_{i0} < \bar{G}$ and the first-order condition for m_i holds with equality.

Next, we show that we are in the case with debt at one of the corners, i.e. $b_i = \tau\chi$ or $b_i = 0$. To see this, we first show that if $b_i \leq \tau\chi(1-\gamma)$ then $b_i = 0$. Suppose the contrary. Substituting (D.3) into (D.4) gives

$$\begin{aligned}&(1-\phi+\phi\theta)\,\phi\theta\frac{\partial w_i(m_i,m_{-i})}{\partial m_i}\left[\log(\tau-b_i)-\log(\tau(1-\gamma)-b_i)\right]\\ &\geq (1-\phi+\phi\theta w_i(m_i,m_{-i}))\frac{1}{\tau-b_i}+\phi\theta(1-w_i(m_i,m_{-i}))\frac{1}{\tau(1-\gamma)-b_i}\end{aligned} \tag{D.9}$$

The left-hand side grows at most at rate $-\log(1-\gamma)$ as γ approaches 1. The right-hand side grows at least as $\frac{1}{1-\gamma}$ as γ approaches one. Hence, there exists $\gamma_2 < 1$, such that if $\gamma > \gamma_2$ the right-hand side exceeds the left-hand side. We must hence have debt at the corner $b_i = 0$.

Next, consider the case $b_i \in (\tau\chi(1-\gamma), \tau\chi]$. Since $\phi\theta\tau\chi < 4$ and $w_1 w_2 \leq \frac{1}{4}$, the numerator on the left-hand side of (D.5) is positive, so

$$g_{i0} = \frac{1-\phi\theta w_1 w_2 b_i}{\phi\theta w_1 w_2\left[\log(\tau-b_i)-\log(\tau(1-\gamma)(1-\chi))\right]}.$$

Since $-\log(\tau(1-\gamma)(1-\chi))$ becomes arbitrarily large as γ approaches one, there exists a threshold $\gamma_3 < 1$, such that in (D.6) the left-hand side exceeds the right-hand side. It then follows that $b_i = \tau\chi$.

It remains to compare the best choice in the low-debt region, $b_i = 0$, with $b_i = \tau\chi$. Let $V_i^L(0)$ and $V_i^H(\tau\chi)$ denote the values of the two programs at these debt levels, each maximized over m_i given m_{-i}, with associated non-military consumption g_{i0}^L, g_{i0}^H, and winning probabilities w_i^L, w_i^H.

Collecting the date 1 terms,

$$
\begin{aligned}
V_i^L(0) &= \log g_{i0}^L + (1-\phi+\phi\theta)\log\tau + \phi\theta\left(1-w_i^L\right)\log(1-\gamma), \\
V_i^H(\tau\chi) &= \log g_{i0}^H + (1-\phi+\phi\theta)\log\left(\tau(1-\chi)\right) + \phi\theta\left(1-w_i^H\right)\log(1-\gamma),
\end{aligned}
$$

so that

$$
V_i^H(\tau\chi) - V_i^L(0) = \log\frac{g_{i0}^H}{g_{i0}^L} + (1-\phi+\phi\theta)\log(1-\chi) + \phi\theta\left(w_i^H - w_i^L\right)\left[-\log(1-\gamma)\right]. \qquad \text{(D.10)}
$$

The first-order conditions (D.3) at $b_i = 0$ and (D.5) at $b_i = \tau\chi$ give

$$
g_{i0}^L = \frac{1}{\phi\theta w_1^L w_2^L\left[-\log(1-\gamma)\right]}, \qquad g_{i0}^H = \frac{1-\phi\theta w_1^H w_2^H \tau\chi}{\phi\theta w_1^H w_2^H\left[-\log(1-\gamma)\right]},
$$

so the first term of (D.10) is bounded with $\underline{\omega} \le w_1 w_2 \le \frac{1}{4}$ and $\phi\theta\tau\chi < 4$. For the third term, both g_{i0}^L and g_{i0}^H vanish as $\gamma \to 1$, and $q_i^H \ge 1-\phi$, so the date 0 budget constraints give

$$
m_i^H - m_i^L = q_i^H\tau\chi - \left(g_{i0}^H - g_{i0}^L\right) \ge \tfrac{1}{2}(1-\phi)\tau\chi > 0
$$

for γ large enough. Since $\frac{\partial w_i}{\partial m_i} = w_1 w_2$ is bounded away from zero, $w_i^H - w_i^L$ is bounded away from zero, and the last term of (D.10) diverges as $\gamma \to 1$. Hence there exists $\gamma_4 < 1$ such that $V_i^H(\tau\chi) > V_i^L(0)$ for all $\gamma > \gamma_4$. Setting $\gamma' = \max\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ then completes the proof. ■

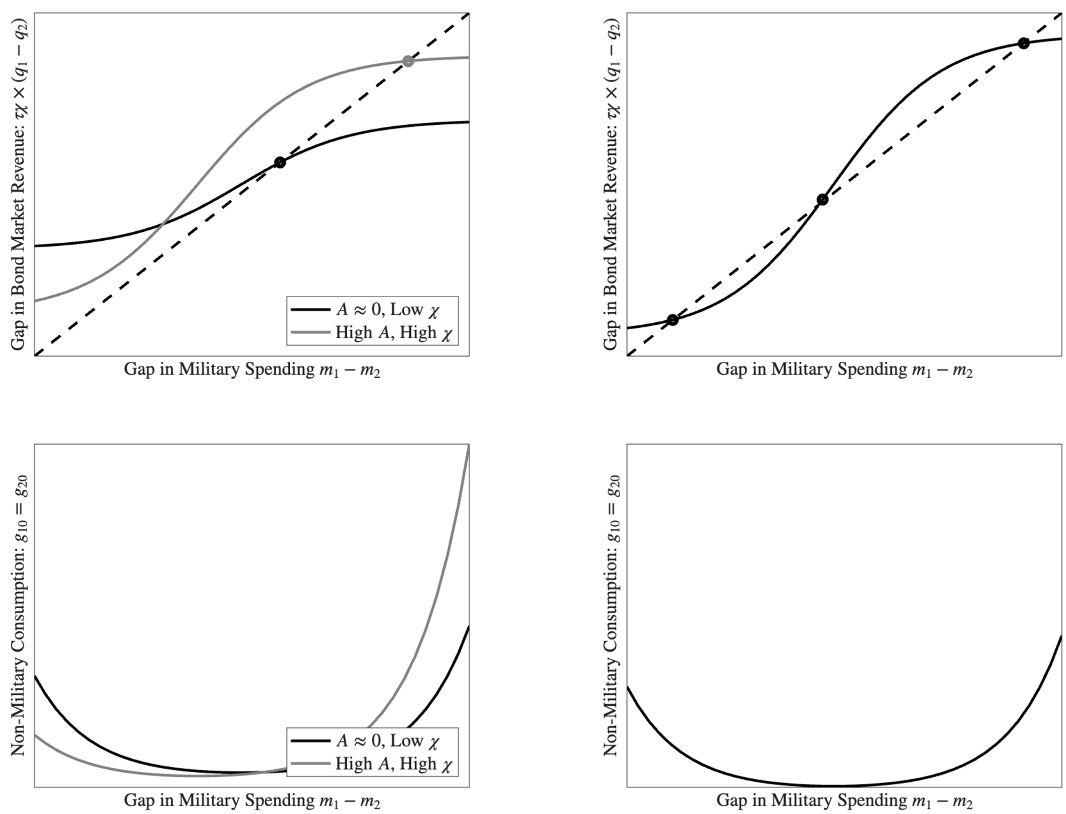
Figure D.1 provides a visual illustration of equilibria with interior non-military consumption and debt at the corner. The top two panels illustrate the equilibrium condition for the military balance from Corollary D.1. We already saw that the equilibrium condition for the military balance is identical to the condition in the main paper, and the top panels in Figure D.1 are hence also identical to Figure 3 in the main paper. The predictions for uniqueness and multiplicity from the linear model therefore are unchanged in the model with concave utility.

The bottom two panels are new and show the optimally chosen non-military consumption as a function of the military balance, as in Proposition D.1. Because both countries optimally choose the same level of non-military consumption, only one line per case is visible. We see that optimal non-military consumption tends to be lowest in the model when the military powers are relatively balanced. Intuitively, with a military rivalry between roughly similar powers, the return from additional military investment is highest, similar to an arms race between two close powers. Conversely, when one country is clearly militarily dominant, there is relatively less benefit to additional military spending for both the dominant country and the non-dominant country. With higher debt capacity, military spending has the additional benefit of raising bond market revenue, shifting down the optimal level of non-military consumption. Despite these interesting implications for non-military consumption, the top panels are identical to the main paper, showing that the equilibria for the military balance and the borrowing cost advantage are robust.

Figure D.1: Equilibria in Two-Period Model with Concave Utility and Finite Cost

Panel A: Unique Equilibrium

Panel B: Multiple Equilibria



This figure illustrates the equilibrium condition for the military balance and non-military consumption for the model with concave utility when non-military consumption is interior but debt is at the upper corner. The top panels plot the equilibrium condition (D.8) and are analogous to Figure 3 in the main paper. The bottom panels plot non-military consumption as a function of the military balance as in equation (D.7).

E Alternative Contest Functions

While our main results rely on the difference contest function of Hirshleifer (1989), the main results on complementarity and multiplicity are robust to assuming one of the other most popular functional forms, i.e. a ratio contest function. We show robustness in the two-period model.

E.1 Ratio Contest Function

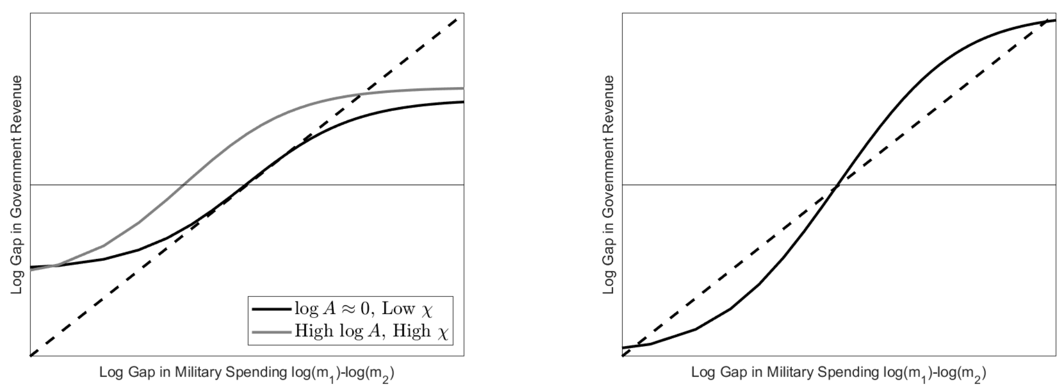
We start with a ratio contest function, where we now assume

$$w_1(m_1, m_2) = F\left(\frac{A \times m_1}{m_2}\right) = \frac{\left(\frac{A \times m_1}{m_2}\right)^{\kappa}}{1 + \left(\frac{A \times m_1}{m_2}\right)^{\kappa}}, \tag{E.1}$$

for some constant κ.

Figure E.1: Equilibria in Two-Period Model with Ratio Contest Function

Panel A: Unique Equilibrium **Panel B:** Multiple Equilibria



This figure is analogous to Figure 3 in the main paper, but it uses a ratio contest function (E.1). The difference in log military capacity $\log m_1 - \log m_2$ is depicted on the x-axis, and the gap in log government revenue $\log(\tau + q_1 b_1) - \log(\tau + q_2 b_2)$ is depicted on the y-axis. An equilibrium is defined by the intersection of the solid line and the dotted 45-degree line. Panel A depicts two examples of unique equilibria. Panel B depicts an example of multiple equilibria. In Panel A, the solid black line represents the gap in log government revenue under a low exogenous military advantage (i.e. $\log A \approx 0$). Debt capacity is sufficiently low that the slope of the curve is always less than one. The solid gray line represents the log gap in government revenue under a high exogenous military advantage $\log A >> 0$ and high debt capacity. The maximum slope of this curve exceeds one, but the exogenous military advantage A is large enough to ensure equilibrium uniqueness. Panel B depicts the gap in log government revenue under a small exogenous military advantage (i.e. $\log A \approx 0$) and high debt capacity, so that the maximum slope of this curve exceeds one and there are multiple equilibria.

We show numerically that the results from the main paper carry over. We start from the equilibrium military investment

$$m_i = \tau + \tau\chi \times (1 - \phi + \phi\theta w_i(m_i, m_{-i})) . \tag{E.2}$$

Taking the difference in log military expenditures gives

$$\begin{aligned}\log m_1 - \log m_2 &= \log\left(\tau + \tau\chi \times \left(1 - \phi + \phi\theta F\left(\log(A) + \log(m_1) - \log(m_2)\right)\right)\right) \\ &\quad - \log\left(\tau + \tau\chi \times \left(1 - \phi + \phi\theta\left(1 - F\left(\log(A) + \log(m_1) - \log(m_2)\right)\right)\right)\right)\end{aligned} \tag{E.3}$$

where with an abuse of notation, we have written the contest function F as a function of $\log A + \log m_1 - \log m_2$. Equation (E.3) is an equilibrium condition for the log difference in military capacity. We plot this equilibrium condition numerically in Figure E.1. It shows that, as in our baseline case, there are two ways to obtain a unique equilibrium. Either if the slope of the revenue curve is low (such as when χ is low) or when one country has a strong exogenous advantage ($\log A \gg 0$). Panel B shows that multiple equilibria obtain when the exogenous advantage is small (now $\log A \approx 0$) and when the slope of the revenue curve is high (χ above a threshold). The similarity with Figure 3 in the main paper hence shows that the features of the two-period model are very similar when we assume a ratio contest function, except that the level of military capacity now needs to be replaced with logs.

F Details of Numerical Illustration

F.1 Additional Assumptions

We set $\tau = 1$ and interpret χ as debt capacity in dollars. It is useful to allow for a factor for the effectiveness of military spending, α. The parameter α can be thought as pinning down the units of our model, since otherwise it is not clear whether m_{it} is measured in dollars, billions of dollars, hundreds of billions, etc. We also allow for a discount factor β, so we can talk about the natural real rate $\bar{r}_f = -\log\beta$. In the quantitative exercise, we set $A = 0$ throughout, i.e. no exogenous military advantage.

These additional assumptions mean that the contest function becomes

$$w(m_i, m_{-i}) = F(\alpha(m_i - m_{-i})), \tag{F.1}$$

and the international investors maximize

$$E\left[c_{i0} + \kappa c_{i1}\right], \ \kappa = \{\beta, \theta \times \beta\} \tag{F.2}$$

F.2 Multiplicity and Fragility Thresholds for Numerical Illustration

Then, the proof of Proposition 2 needs to be modified as follows. The bond price equals

$$\begin{aligned} q_{it}(m_{it}, m_{-it}) &= \beta\left(1 - \phi + \phi\theta w(m_{it}, m_{-it})\right), && \text{(F.3)} \\ &= \beta\left(1 - \phi + \phi\theta F(\alpha(m_{it} - m_{-it})\right). && \text{(F.4)} \end{aligned}$$

A steady state is a zero of the function

$$h(z) = \delta z - \chi\phi\theta\beta\tanh\left(\frac{\alpha z}{2}\right) \tag{F.5}$$

The derivative with respect to z equals

$$h'(z) = \delta - \chi\phi\theta\beta\alpha\frac{1}{2}\left(1 - \tanh^2\left(\frac{\alpha z}{2}\right)\right). \tag{F.6}$$

Table F.1: Numerical Illustration Parameter Values

Parameter		Value	Source
Period length (years)	T	6	Avg. US debt maturity 2023
War probability (% p.a.)	ϕ	1.7%	Barro (2006)
Depreciation rate (% p.a.)	δ	8.0%	Cooley and Prescott (1995)
Discount factor	β	0.99	r-star from Laubach and Williams (2003)
Investor disaster risk aversion	ψ	4	Barro (2006)
Disaster economic contraction	b	35.0%	Barro (2006)
War discount factor	θ	5.60	Implied $\equiv (1-b)^{-\psi}$

All parameter values are in annualized units. The period war probability, depreciation rate, and discount factor are computed as $\phi_T = 1-(1-\phi)^T$, $\delta_T = 1-(1-\delta)^T$, and $\beta_T = \beta^T$.

From the proof of Proposition 2, it then follows that

$$\chi' = \frac{2\delta}{\phi\theta\alpha\beta}, \tag{F.7}$$

To derive the fragility threshold, note that the equilibrium mapping can be written as

$$z_{t-1} = H(z_t) \quad = \quad \frac{z_t - \chi\phi\theta\beta\tanh\left(\frac{\alpha z_t}{2}\right)}{1-\delta}. \tag{F.8}$$

The derivative is given by

$$H'(z_t) \quad = \quad \frac{1 - \frac{\chi\phi\theta\alpha\beta}{2}\left(1-\tanh^2\left(\frac{\alpha z_t}{2}\right)\right)}{1-\delta} \tag{F.9}$$

The mapping H has a downward-sloping portion if and only if $\chi > \chi''$ where the fragility threshold is now given by

$$\chi'' \quad = \quad \frac{2}{\phi\theta\alpha\beta}. \tag{F.10}$$

F.3 Parameter Values

We set the war probability to 1.7% per year, the consumption decline conditional on a disaster to 35%, and CRRA risk aversion over the disaster state to $\psi = 4$. The implied war discount factor then equals $\theta = (1-b)^{-\psi} = 5.60$, where b is investors' consumption decline in a disaster. The length of one period is set to 6 years to match the maturity of U.S. government debt, and the depreciation rate is set to a conventional value of 8% annualized (Cooley and Prescott (1995)). We allow for a time discount factor, which is set to $\beta = 0.99$ in annualized units. The exogenous military advantage, A, is set to zero throughout this section. Parameter values needed for the symmetric country plots are listed in Table F.1.

For the asymmetric model plots, we additionally need the effectiveness of military spending on the probability of winning a war, α. We match this to recent estimates by Federle, Rohner and Schularick (2024). Using commodity price shocks, they estimate that an additional 10% of resources (in GDP units) increases government revenues by 5% and increases the probability of winning a war by 3.2%. Their sample consists of 135 countries since 1977. In the model

$$\frac{dw_i(m_i, m_{-i})}{dm_i} = \frac{\alpha}{4}\left(1 - \tanh^2\left(\frac{\alpha(m_1 - m_2)}{2}\right)\right) \leq \frac{\alpha}{4}. \tag{F.11}$$

Hence, if we have two contestants with relatively even winning probabilities, this implies

$$\alpha = 4 \times \frac{dw_i(m_i, m_{-i})}{dm_i} = 4 \times \frac{3.2}{5} = 2.56. \tag{F.12}$$

With this value for α, the implied baseline multiplicity threshold with symmetric countries, $\bar{\chi}'$ equals 60% of GDP and the baseline fragility threshold equals 152% of GDP.

F.4 Symmetric Countries Plots

We plot the multiplicity and fragility thresholds relative to a baseline value, which allows us to inspect how the threshold functions vary without having to take a stance on the value of α, which is particularly hard to quantify. In particular, we plot χ' and χ'' scaled by the multiplicity threshold at baseline parameter values $\underline{\phi}, \underline{\theta}, \underline{\beta}, \underline{\delta}$, i.e. we plot the quantities

$$\frac{\chi'(\phi, \theta, \beta, \delta, \alpha)}{\bar{\chi}'\left(\underline{\phi}, \underline{\theta}, \underline{\beta}, \underline{\delta}, \alpha\right)} = \frac{\delta}{\phi\theta\beta} / \left(\frac{\underline{\delta}}{\underline{\phi}\underline{\theta}\underline{\beta}}\right), \tag{F.13}$$

and

$$\frac{\chi''(\phi, \theta, \beta, \delta, \alpha)}{\bar{\chi}'\left(\underline{\phi}, \underline{\theta}, \underline{\beta}, \underline{\delta}, \alpha\right)} = \frac{1}{\phi\theta\beta} / \left(\frac{\underline{\delta}}{\underline{\phi}\underline{\theta}\underline{\beta}}\right) \tag{F.14}$$

To plot these thresholds against investor risk aversion ψ or the natural real rate on the x-axis, we use $\theta = (1-b)^{-\psi}$ and $\bar{r}_f = -\log\beta$.

F.5 Plots with Asymmetric Debt Capacity

Now we consider the dynamic model with asymmetric ability to pledge future debt repayments, $\chi_1 > \chi_2$. For completeness, we also allow for $\tau \neq 1$, though we will show that this drops out for the plots of multiplicity/fragility thresholds relative to the equal debt capacity benchmark. The equilibrium dynamics for countries 1 and 2 are

$$\begin{aligned} m_{1t} &= \tau + (1-\delta)\, m_{1t-1} - \tau\chi_1(1-\beta) - \tau\chi_1\beta \times (\phi - \phi\theta F\left(\alpha\left(m_{1t} - m_{2t}\right)\right)) \\ m_{2t} &= \tau + (1-\delta)\, m_{2t-1} - \tau\chi_2(1-\beta) - \tau\chi_2\beta \times (\phi - \phi\theta\left(1 - F\left(\alpha\left(m_{1t} - m_{2t}\right)\right)\right)) \end{aligned}$$

Taking the difference between countries 1 and 2 yields

$$\begin{aligned} \mu_t - (1-\delta)\,\mu_{t-1} &= (-\phi\beta - (1-\beta))\,\tau\left(\chi_1 - \chi_2\right) + \tau\chi_1 \times \phi\theta\beta F(\alpha\mu_t) - \tau\chi_2 \times \phi\theta\beta\left(1 - F(\alpha\mu_t)\right), \\ &= \left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)\tau\left(\chi_1 - \chi_2\right) + \tau\frac{\chi_1 + \chi_2}{2} \times \phi\theta\beta(2F(\alpha\mu_t) - 1). \end{aligned} \tag{F.15}$$

A steady-state must be a zero of the function

$$h(z) = \frac{\delta}{\alpha}\alpha z - \frac{\delta}{\alpha} \times \frac{\alpha}{\delta}\left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)\tau\left(\chi_1 - \chi_2\right) - \tau\frac{\chi_1 + \chi_2}{2} \times \phi\theta\beta\tanh\left(\frac{\alpha z}{2}\right). \tag{F.16}$$

Assuming that $\frac{\tau}{\delta}\frac{\chi_1+\chi_2}{2} \times \phi\theta\beta\alpha > 2$, there is a unique steady state if and only if

$$\tilde{h}\left(\frac{\tau}{\delta}\frac{\chi_1+\chi_2}{2} \times \phi\theta\beta\alpha\right) < \frac{\alpha}{\delta}\left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)\tau\left(\chi_1 - \chi_2\right). \tag{F.17}$$

The multiplicity threshold for χ_1 at a given ratio χ_2/χ_1 then must satisfy

$$\tilde{h}\left(\chi_1\alpha\frac{\tau}{\delta}\frac{1+\chi_2/\chi_1}{2}\phi\theta\beta\right) - \chi_1\alpha\frac{\tau}{\delta}\left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)(1-\chi_2/\chi_1) \quad = \quad 0. \tag{F.18}$$

We plot the multiplicity threshold scaled by the baseline with equal debt capacity, i.e.

$$\frac{\chi_1'}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} = \frac{\alpha\tau\chi_1'}{\frac{2\delta}{\phi\theta\beta}}. \tag{F.19}$$

We can find the product $\alpha\tau\chi_1'$ numerically as the zero of (F.18). Note that the solution for the product $\alpha\tau\chi_1'$ is independent of α and τ, so the ratio (F.19) is also independent of α and τ. We verify numerically that there is only one zero, so this zero gives the unique threshold such that there are multiple steady states when χ_1 exceeds the threshold.

Following the proof of Proposition 3, the fragility threshold for χ_1 for a given ratio χ_2/χ_1 is given by

$$\chi_1'' = \frac{2}{\tau\phi\theta\beta\alpha}\frac{1}{\frac{1}{2}\left(1+\frac{\chi_2}{\chi_1}\right)}, \tag{F.20}$$

and we plot this fragility threshold scaled by the baseline multiplicity threshold:

$$\frac{\chi_1''}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} \quad = \quad \frac{1}{\frac{\delta}{2}\left(1+\frac{\chi_2}{\chi_1}\right)}. \tag{F.21}$$

Note that this second ratio is also independent of α and τ, so we do not need to take a stance on these quantities to understand how the multiplicity and fragility thresholds vary compared to the benchmark where both countries have equal access to international financial markets.

F.6 Plots with Asymmetric Country Size

Now, consider the case with $\tau_1 > \tau_2$ and equal debt capacity $\chi_1 = \chi_2 = \chi$. The dynamic equation for country i's budget constraint becomes

$$m_{it} \quad = \quad \tau_i + (1-\delta)\,m_{it-1} + (\beta-1)\chi\tau_i - \tau_i\chi \times \phi\beta\left(1-\theta w_i(m_{it}, m_{-it})\right)$$

Taking the difference between countries 1 and 2 gives

$$\begin{aligned}\mu_t - (1-\delta)\,\mu_{t-1} \quad &= \quad (\tau_1-\tau_2)\left(1-\chi(1-\beta)-\chi\times\phi\beta\right) + \tau_1\chi\times\phi\theta\beta F(\alpha\mu_t) - \tau_2\chi\times\phi\theta\beta\left(1-F(\alpha\mu_t)\right), \\ &= \quad (\tau_1-\tau_2)\left(1-\chi(1-\beta)+\chi\times\phi\beta\left(\frac{\theta}{2}-1\right)\right) + \frac{\tau_1+\tau_2}{2}\chi\times\phi\theta\beta\tanh\left(\frac{\alpha\mu_t}{2}\right)\end{aligned} \tag{F.22}$$

Here, we have used that $2F(\alpha\mu_t) - 1 = \tanh\left(\frac{\alpha\mu_t}{2}\right)$. Assume that the war discount factor θ is high enough that the intercept in (F.22) is positive. A steady state is a zero of the following function

$$h(z) = \frac{\delta}{\alpha}\left(\alpha z - \frac{\alpha}{\delta}(\tau_1 - \tau_2)\left(1 - \chi(1-\beta) + \chi \times \phi\beta\left(\frac{\theta}{2} - 1\right)\right)\right) - \frac{\tau_1 + \tau_2}{2}\chi \times \phi\theta\beta \tanh\left(\frac{\alpha z}{2}\right).$$

Comparing to equation (A.12) in the proof of Proposition 2 shows that there exists a unique steady state with country 1 dominating if and only if $\frac{\alpha}{\delta}\frac{\tau_1+\tau_2}{2}\chi \times \phi\theta\beta < 2$ or if

$$\tilde{h}\left(\frac{\frac{\tau_1+\tau_2}{2}\chi \times \phi\theta\beta\alpha}{\delta}\right) \quad < \quad \frac{\alpha}{\delta}(\tau_1 - \tau_2)\left(1 - \chi(1-\beta) + \chi \times \phi\beta\left(\frac{\theta}{2} - 1\right)\right). \tag{F.23}$$

We plot the zero of (F.23) scaled by the multiplicity threshold when both countries have equal tax revenue, i.e.

$$\frac{\chi'(\tau_1 \neq \tau_2)}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} \tag{F.24}$$

The fragility threshold is given by

$$\chi'' \quad = \quad \frac{2}{\phi\beta\theta\alpha\frac{\tau_1+\tau_2}{2}}, \tag{F.25}$$

showing that it decreases as τ_1 increases. We plot

$$\frac{\chi''(\tau_1 \neq \tau_2)}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} \tag{F.26}$$

References

Barro, Robert J. 2006. "Rare Disasters and Asset Markets in the Twentieth Century." *Quarterly Journal of Economics*, 121(3): 823–866.

Caldara, Dario, and Matteo Iacoviello. 2022. "Measuring Geopolitical Risk." *American Economic Review*, 112(4): 1194–1225.

Cooley, Thomas F, and Edward C Prescott. 1995. "Economic growth and business cycles." *Frontiers of business cycle research*, 1: 1–38.

Du, Wenxin, Joanne Im, and Jesse Schreger. 2018. "The U.S. Treasury Premium." *Journal of International Economics*, 112: 167–181.

Du, Wenxin, Ritt Keerati, and Jesse Schreger. 2025. "Decoupling Dollar and Treasury Privilege." Board of Governors of the Federal Reserve System International Finance Discussion Paper 1427.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization*, 49(3): 379–414.

Federle, Jonathan, Dominic Rohner, and Moritz Schularick. 2024. "Who wins wars?" *Available at SSRN 5093393.*

Hirshleifer, Jack. 1989. "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success." *Public Choice*, 63(2): 101–112.

Hu, Grace Xing, Jun Pan, and Jiang Wang. 2013. "Noise as Information for Illiquidity." *Journal of Finance*, 68(6): 2341–2382.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. "The Aggregate Demand for Treasury Debt." *Journal of Political Economy*, 120(2): 233–267.

Laubach, Thomas, and John C Williams. 2003. "Measuring the natural rate of interest." *Review of Economics and Statistics*, 85(4): 1063–1070.

Morgenthau, Hans J. 1948. *Politics Among Nations: The Struggle for Power and Peace.* New York:Alfred A. Knopf.

Nagel, Stefan. 2016. "The Liquidity Premium of Near-Money Assets." *Quarterly Journal of Economics*, 131(4): 1927–1971.

Organski, A. F. K., and Jacek Kugler. 1980. *The War Ledger.* Chicago:University of Chicago Press.

Powell, Robert. 1993. "Guns, Butter, and Anarchy." *American Political Science Review*, 87(1): 115–132.

Powell, Robert. 1999. *In the Shadow of Power: States and Strategies in International Politics.* Princeton University Press.

Sample, Susan G. 1997. "Arms Races and Dispute Escalation: Resolving the Debate." *Journal of Peace Research*, 34(1): 7–22.

Wallace, Michael D. 1979. "Arms races and escalation: Some new evidence." *Journal of Conflict Resolution*, 23(1): 3–16.

Waltz, Kenneth N. 1979. *Theory of International Politics.* Reading, MA:Addison-Wesley.